Alfvén soliton and emitted radiation in the presence of perturbations

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Soliton perturbation theory based on the inverse scattering transform method is applied to the derivative nonlinear Schrödinger equation, which describes nonlinear Alfvén waves propagating quasiparallel to the external magnetic field. Radiative effects are considered. Spectral distributions of the emitted energy and magnetic helicity rates (in the wave number domain) are calculated analytically. Several forms of perturbations are considered, including the finite electric conductivity, the effect of resonant particles (nonlinear Landau damping), and the influence of the random inhomogeneity of the plasma density. The space structure of the radiative field is determined for a perturbation in the form of the finite electric conductivity.

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I. INTRODUCTION

It is well known [1] that the evolution of small-amplitude nonlinear Alfvén waves propagating parallel or at a small angle to the background magnetic field in a low-β (the ratio of kinetic to magnetic pressure) plasma is governed by the derivative nonlinear Schrödinger (DNLS) equation. Recently it was shown [2] that the DNLS equation also describes large-amplitude magnetohydrodynamic waves in a high-β plasma, propagating at an arbitrary angle to the ambient magnetic field. Originally, the DNLS equation was derived by Rogister [3] from the Vlasov kinetic equation and then by Mjølhus [4] and Mio *et al.* [5] for a cold plasma. Later, Spangler and Sheerin [6] and Sakai and Sonnerup [7] generalized these results to a finite-β plasma. A comprehensive review of the theory of small-amplitude Alfvén waves based on the DNLS equation has been given by Mjølhus and Hada [1].

Kaup and Newell showed [8] that the DNLS equation was solvable by the inverse scattering transform (IST) method and it admitted N-soliton solutions. Then it was shown [9] that the DNLS equation is a completely integrable Hamiltonian system and the corresponding "action-angle" variables were explicitly calculated.

In reality, additional terms are often present in the DNLS equation. They can include effects of dissipation, influence of external forces, inhomogeneity of the plasma density, etc. [1,10–12]. These terms violate the integrability, but being small in many important practical cases, they can be taken into account by perturbation theory. For Alfvén waves propagating along the static magnetic field B_0, which is directed along the z axis, the perturbed DNLS equation can be written in the following normalized form:

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial z^2} + is\frac{\partial}{\partial z}(|u|^2 u) = p[u,u^*], \tag{1}$$

where $u(z,t)$ is the normalized transverse magnetic field perturbation $u=(B_x+iB_y)/(2\sqrt{|1-\beta|}B_0)$. The field $u(z,t)$ and its conjugate $u^*(z,t)$ correspond to the right- and left-hand-side circular polarizations, respectively. Time t and space coordinate z are normalized in terms of $1/\omega_{ci}$ and v_A/ω_{ci}, respectively, where $\omega_{ci}=eB_0/Mc$ is the ion-cyclotron frequency and $v_A=B_0/\sqrt{4\pi n_0 M}$ is the Alfvén velocity. The parameter $\beta=v_s^2/v_A^2$ is the ratio of kinetic to magnetic pressure, and v_s is the sound speed. The sign s of the nonlinear term in Eq. (1) corresponds to $s=\text{sgn}(1-\beta)$. Equation (1) is written in the frame moving with the Alfvén velocity v_A. The perturbation is represented by the term $p[u,u^*]$.

The most powerful perturbative technique, which fully uses the natural separation of the discrete and continuous (i.e., solitonic and radiative) degrees of freedom of the integrable equations, is based on the IST [13–15]. For the DNLS equation a perturbation theory using the IST was developed by Wyller and Mjølhus [11]. They applied their formalism to a study of the influence of dissipative perturbations (finite Ohmic resistance [11] and nonlinear Landau damping [12]) on a single Alfvén soliton described by the DNLS equation. The authors of Ref. [11] derived the corresponding adiabatic equations which are evolution equations for discrete-spectrum (solitonic) scattering data. In the frame of the IST, they calculated also the reflection coefficient which describes the excitation of the radiative degrees of freedom [11,12]. That allowed an explanation of why the adiabatic ansatz fails to produce a good approximation of the perturbational dynamics in the anomalous regime (i.e., when the soliton moves in the positive z direction) of the Alfvén solitons. This result was confirmed also by numerical analysis [12]. Note that adiabatic equations for soliton parameters can be obtained, generally speaking, with the aid of integrals of motion, without using the IST. On the other hand, only the perturbative technique based on the IST allows one to take into account the excitation of continuous (radiative) degrees of freedom, which gives rise to qualitatively new effects in one-soliton dynamics. These effects include, in particular, perturbation-induced emission of radiation by a soliton, long-range corrections to the soliton's shape ("tails"), and the generation of new (secondary) solitons [14,15,17]. In addition, the IST formalism allows one to obtain a criterion of applicability of the adiabatic approach [11,12].

The aim of this paper is to consider radiative effects under the influence of perturbations on the Alfvén soliton. We analyze the evolution equation for the continuous scattering data

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and obtain spectral distributions (in the wave number domain) of the energy and magnetic helicity emission rate for several forms of perturbation. For a perturbation in the form of the finite electric conductivity we determine the space structure of the radiative field.

The paper is organized as follows. In Sec. II we review a theory of the scattering transform for the linear eigenvalue problem associated with the DNLS equation. In Sec. III integrals of motion are written in terms of the discrete and continuous scattering data and the relation between a spectral parameter of the eigenvalue problem and wave number of the radiation is established. Dissipative perturbations (finite electric resistance and nonlinear Landau damping) are considered in Sec. IV, and random fluctuations of the plasma density are considered in Sec. V. The conclusion is made in Sec. VI.

II. INVERSE SCATTERING TRANSFORM FOR THE DNLS EQUATION

In this section we review the theory of the scattering transform for the DNLS equation, following (with slight modifications) Refs. [8,9]. Equation (1) with $p[u,u^*]=0$ can be written as the compatibility condition

$$\partial_t U - \partial_x V + [U,V] = 0 \tag{2}$$

of two linear matrix equations (Kaup-Newell pair) [8]

$$\partial_z M(z,t,\lambda) = UM(z,t,\lambda), \tag{3}$$

$$\partial_t M(z,t,\lambda) = VM(z,t,\lambda), \tag{4}$$

where λ is a spectral parameter and

$$U = -i\lambda^2\sigma_3 + \lambda Q, \quad \text{with } Q = \begin{pmatrix} 0 & u \\ -u^* & 0 \end{pmatrix}, \tag{5}$$

$$V = -2i\lambda^4\sigma_3 + 2\lambda^3 Q - i\lambda^2 Q^2\sigma_3 + \lambda Q^3 - i\lambda Q_z\sigma_3, \tag{6}$$

where

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is the Pauli matrix. The Jost solutions $M^{\pm}(z,\lambda)$ of Eq. (3) for real λ^2 (i.e., fundamental solutions) and for some fixed t are defined by the boundary conditions

$$M^{\pm}(z,\lambda) = \exp(-i\lambda^2\sigma_3 z) + o(1), \quad \text{as } z \to \pm\infty. \tag{7}$$

The matrix Jost solutions (7) can be represented in the form $M^- = (\varphi, -\bar{\varphi})$ and $M^+ = (\bar{\psi}, \psi)$, where φ and ψ are independent vector columns. The scattering matrix $S(\lambda)$ relates the two fundamental solutions M^- and M^+:

$$M^-(z,\lambda) = M^+(z,\lambda)S(\lambda). \tag{8}$$

The scattering coefficients are defined by

$$\varphi = a\bar{\psi} + b\psi, \tag{9}$$

$$\bar{\varphi} = -\bar{a}\psi + \bar{b}\bar{\psi}, \tag{10}$$

so that the scattering matrix is

$$S(\lambda) = \begin{pmatrix} a(\lambda) & -\bar{b}(\lambda) \\ b(\lambda) & \bar{a}(\lambda) \end{pmatrix}, \tag{11}$$

where $a\bar{a}+b\bar{b}=1$. The vector functions $\varphi(z,\lambda)$ and $\psi(z,\lambda)$ and the coefficient $a(\lambda)$ turn out to be analytically continuable to $\mathrm{Im}\,\lambda^2>0$. As follows from Eq. (3), the following important symmetry properties are valid:

$$\bar{\varphi}(\lambda) = -i\sigma_2\varphi^*(\lambda^*), \quad \bar{\psi}(\lambda) = i\sigma_2\psi^*(\lambda^*). \tag{12}$$

The zeros $\lambda_j^2=\xi_j+i\eta_j$ $(j=1,\ldots,N)$ of the function $a(\lambda)$ in the region of its analiticity $\mathrm{Im}\,\lambda^2>0$ give the discrete spectrum of the linear problem (3) and correspond to solitons. Under this, the functions $\varphi(z,\lambda_j)$ and $\psi(z,\lambda_j)$ are linearly dependent:

$$\varphi(z,\lambda_j) = b_j\psi(z,\lambda_j), \quad \bar{\varphi}(z,\lambda_j^*) = -b_j^*\bar{\psi}(z,\lambda_j^*). \tag{13}$$

The Jost coefficients $a(\lambda)$ and $b(\lambda)$ with real λ^2 constitute the continuous spectrum scattering data, and the set of complex numbers λ_j and b_j are the discrete spectrum scattering data. The time evolution of these scattering data turns out to be trivial:

$$a(\lambda,t) = a(\lambda,0), \quad b(\lambda,t) = b(\lambda,0)\exp(4i\lambda^4 t), \tag{14}$$

$$\lambda_j(t) = \lambda_j(0), \quad b_j(t) = b_j(0)\exp(4i\lambda^4 t). \tag{15}$$

A nonsoliton (radiative) part of the field is completely defined by the continuous spectrum—namely, by the so-called reflection coefficient $r(\lambda)=b(\lambda)/a(\lambda)$ with $\mathrm{Im}\,\lambda^2=0$. The field $u(z,t)$ is expressed in terms of the scattering data and Jost solutions of Eq. (3) as follows [9]:

$$u(z,t) = \frac{1}{\pi}\int_\Gamma (r\psi_1^2 + \bar{r}\bar{\psi}_1^2)d\lambda + \frac{4}{i}\sum_{j=1}^{N}(c_j\psi_{1,j}^2 + c_j^*\bar{\psi}_{1,j}^2), \tag{16}$$

where $\bar{r}(\lambda)\equiv\bar{b}(\lambda)/\bar{a}(\lambda)=r^*(\lambda^*)$, $c_j=b_j/a_j'$ with $a_j'=da/d\lambda|_{\lambda=\lambda_j}$. The contour Γ consists of lines from $i\infty$ to 0, from $-i\infty$ to 0, from 0 to ∞, and from 0 to $-\infty$. The first term in Eq. (16) is the radiative part of the field, while the second one corresponds to the soliton contribution. If the reflection coefficient $r(\lambda)$ is identically zero, then $u(z,t)$ is an exact (one- or multiple-) soliton solution of the DNLS equation. The reflectionless scattering data with the single $(N=1)$ zero $\lambda_1^2=\xi+i\eta$ of the function $a(\lambda)$ correspond to the one-soliton solution (a single Alfvén soliton)

$$u_s(z,t) = \frac{2\eta}{|\lambda_1|}\frac{\cosh(k_0 y - i\theta)}{\cosh^2(k_0 y + i\theta)}e^{i\phi}, \tag{17}$$

where we have introduced the notations

$$y = z - vt - z_0, \quad \phi = \phi_0 + 8\eta^2 t - 2\xi y, \tag{18}$$

$$k_0 = 2\eta, \quad v = -4\xi, \quad \theta = \arg(\lambda_1). \tag{19}$$

The parameters z_0 and ϕ_0 determine the initial position and initial phase of the soliton. The parameters η and ξ are, up to constant multipliers, the soliton inverse width k_0 and the soliton velocity v, respectively. The soliton solution of Eq. (17) (in somewhat different form) was first obtained by Mjølhus [4] and then rederived [8] with the use of the IST. Explicit expressions for the one-soliton Jost solutions and corresponding scattering data are given in Appendix A.

III. INTEGRALS OF MOTION

Since the DNLS equation is completely integrable, it has an infinite set of integrals of motion. We will consider only three of them: the energy,

$$E=\int_{-\infty}^{\infty}|u|^2dz, \tag{20}$$

Hamiltonian

$$H=\frac{1}{2}\int_{-\infty}^{\infty}\left\{i\left(u^*\frac{\partial u}{\partial z}-u\frac{\partial u^*}{\partial z}\right)-|u|^4\right\}dz, \tag{21}$$

and magnetic helicity, defined as

$$K=\int_{-\infty}^{\infty}\mathbf{A}\cdot\mathbf{B}dz, \tag{22}$$

where the vector potential $\mathbf{A}$ is introduced through the magnetic field $\mathbf{B}$ as $\mathbf{B}=\nabla\times\mathbf{A}$. It was shown [16] that the magnetic helicity K belongs to a hierarchy of nonlocal conservation laws of the DNLS equation. The infinite hierarchy of nonlocal integrals of motion was first introduced in Ref. [9] (see also Ref. [17]). In terms of the field u the (normalized) helicity is written as

$$K=\frac{i}{2}\int_{-\infty}^{\infty}\left(u\int_{\infty}^{z}u^*(z')dz'-u^*\int_{\infty}^{z}u(z')dz'\right)dz. \tag{23}$$

The integrals of motion can be explicitly expressed in terms of the continuous (radiative) and discrete (solitonic) scattering data. In particular, one can write [9]

$$E=-2i\sum_{j=1}^{N}\ln\frac{\lambda_j^2}{\lambda_j^{*2}}+\int_\Gamma\mathcal{E}(\lambda)d\lambda, \tag{24}$$

$$H=4i\sum_{j=1}^{N}(\lambda_j^2-\lambda_j^{*2})+\int_\Gamma\mathcal{H}(\lambda)d\lambda, \tag{25}$$

$$K=-i\sum_{j=1}^{N}\left(\frac{1}{\lambda_j^2}-\frac{1}{\lambda_j^{*2}}\right)+\int_\Gamma\mathcal{K}(\lambda)d\lambda, \tag{26}$$

where

$$\mathcal{E}(\lambda)=\frac{\ln[1+r(\lambda)\bar{r}(\lambda)]}{\pi\lambda}, \tag{27}$$

$$\mathcal{H}(\lambda)=-2\lambda^2\mathcal{E}(\lambda),\quad \mathcal{K}(\lambda)=-\frac{1}{2\lambda^2}\mathcal{E}(\lambda). \tag{28}$$

In Eqs. (24)–(26) the soliton contribution ($\Sigma_{j=1}^{N}$) is separated from that of the radiative component ($\int d\lambda$) of the wave field described by the continuous-spectrum scattering data. The dispersion relation [taking $\sim\exp(-ikz+i\omega t)$] corresponding to the linearized version of Eq. (1) is $\omega(k)=-k^2$, which means the t dependence $\sim\exp(-ik^2t)$. On the other hand, as follows from Eq. (14), in the nonlinear case the t dependence for the continuous spectrum data is $\sim\exp(4i\lambda^4t)$. Moreover, one can show that in the linear limit we have $\psi_1^2\to 0$, while $\bar{\psi}_1^2$ is reduced to $\exp(-2i\lambda^2z-4i\lambda^4t)$ and r^*/λ^* is just the Fourier transform of $u(z,t)$.This reflects the general property of the IST (see, for example, Ref. [18]): in the linear limit it is equivalent to the usual Fourier method. Then, considering the radiative component as a superposition of free waves governed by the linear Schrödinger equation, one can conclude that the spectral parameter λ is connected to the wave number of the emitted quasilinear waves k by the relation

$$k=2\lambda^2, \tag{29}$$

where λ^2 is real (continuous spectrum). The integral over the contour Γ in Eqs. (16) and (24)–(26) can be transformed into the one over the real axis k. If only one soliton is present in the general field (i.e., $N=1$ and $\lambda_1^2=\xi+i\eta$), we have

$$E=8\theta+\int_{-\infty}^{\infty}\mathcal{E}_{rad}(k)dk, \tag{30}$$

$$H=-8\eta+\int_{-\infty}^{\infty}\mathcal{H}_{rad}(k)dk, \tag{31}$$

$$K=-\frac{2\eta}{\xi^2+\eta^2}+\int_{-\infty}^{\infty}\mathcal{K}_{rad}(k)dk, \tag{32}$$

where

$$\mathcal{E}_{rad}(k)=\frac{\ln[1+\operatorname{sgn}k|r(k)|^2]}{\pi k}, \tag{33}$$

$$\mathcal{H}_{rad}(k)=-2k\mathcal{E}_{rad}(k),\quad \mathcal{K}_{rad}(k)=-\frac{1}{2k}\mathcal{E}_{rad}(k). \tag{34}$$

The quantities $\mathcal{E}_{rad}(k)$, $\mathcal{H}_{rad}(k)$, and $\mathcal{K}_{rad}(k)$ can be regarded as spectral densities (in the wave number domain) of the energy, Hamiltonian, and magnetic helicity carried by the radiation. Note that, as follows from Eqs. (30)–(32), for the pure soliton solution ($r\equiv 0$) there exists the following interesting relation between soliton energy, Hamiltonian, and magnetic helicity:

$$KH=16\sin^2(E/8). \tag{35}$$

Equation (16) for the general field becomes

$$u(z,t)=\frac{1}{\pi}\int_{-\infty}^{\infty}(R\psi_1^2+R^*\bar{\psi}_1^2)dk+\frac{4}{i}\sum_{j=1}^{N}(c_j\psi_{1,j}^2+c_j^*\bar{\psi}_{1,j}^2), \tag{36}$$

where we have introduced the function $R(k)=r/\lambda$.

IV. DISSIPATIVE PERTURBATIONS IN THE DNLS EQUATION

In this section we investigate radiative effects which arise under the influence of dissipative perturbations on the Alfvén soliton. We will consider two different cases: the collision-dominated case, when finite electric conductivity (and/or ion viscosity) is taken into account in the basic equation (1), and collisionless nonlinear Landau damping. In the first case the perturbation term in Eq. (1) has the diffusive form

$$p = iD\frac{\partial^2 u}{\partial z^2}, \tag{37}$$

where the diffusion coefficient is [20]

$$D = \frac{1}{2}\left(\frac{\eta_1}{\rho_0} + \frac{c^2}{4\pi}\eta_2\right)\frac{\omega_{ci}}{v_A^2}, \tag{38}$$

where η_1 is a coefficient of ion viscosity and η_2 is the resistivity. Expressions for η_1 and η_2 in some limiting cases can be found in Ref. [18]. The conditions under which the diffusive term, Eq. (37), can be considered as a small perturbation are (in the range $v_i \sim v_A$) [20]

$$\omega \ll \nu_i \ll \omega_{ci} \tag{39}$$

or

$$\omega_{ci} \ll \nu_i \ll \omega_{ci}\sqrt{M/m}, \tag{40}$$

where v_i is the ion thermal velocity and ν_i is the ion collision frequency. In the second, collisionless case the perturbation in Eq. (1) is presented by the resonant-particle term [19,20]

$$p = i\frac{C}{\pi}\frac{\partial}{\partial z}\left(uP\int_{-\infty}^{\infty}\frac{|u(z',t)|^2}{z'-z}dz'\right), \tag{41}$$

where P is the symbol of the principal value. Including this kinetic term in the DNLS equation is especially important for the case of $\beta \sim 1$ and the electron-to-ion temperature ratio $T_e/T_i \sim 1$ (for example, in solar wind plasma), when Alfvén waves couple to strongly damped ion-acoustic modes [21,22]. The coefficient C in Eq. (41) depends on the velocity distributions of the particle species. As was pointed out in Refs. [12,19], when restricting the study to waves propagating in plasmas with isotropic Maxwellian-distributed electrons and ions, the effect of resonant particles is weak and can be regarded as a perturbation (i.e., $C \ll 1$) when the Alfvén velocity v_A is much larger than the ion sound velocity v_s (cold plasma with $\beta \ll 1$). Under this,

$$C = \sqrt{\frac{m}{2\pi M}}\frac{v_s}{v_A}\exp\left(-\frac{v_A^2}{2v_e^2}\right), \quad v_A \gg v_s, \tag{42}$$

where v_e is the electron thermal velocity. Adiabatic equations describing the slow evolution of the soliton parameters (width and velocity)—i.e. the discrete spectrum scattering data—under the action of perturbation of the form, Eq. (37) or Eq. (41), were obtained and analyzed in Refs. [11,12]. The adiabatic approximation implies that $r(\lambda)=0$ and an unperturbed instantaneous shape of the soliton is assumed. Now we are interested in radiative effects which are described by the continuous spectrum scattering data—namely, by the reflection coefficient $r(\lambda)$.

In the presence of a perturbation [i.e., $p \neq 0$ in Eq. (1)], the reflection coefficient $r(\lambda)$ is no longer zero, and for small p we have $|r(\lambda)| \ll 1$. Thus, the spectral density of the energy, Eq. (33), can be written as

$$\mathcal{E}_{rad}(k) = \frac{|r(k)|^2}{\pi|k|}. \tag{43}$$

The emission intensity is characterized by its power—i.e., the energy emission rate. The emission power spectral density $W(k) \equiv d\mathcal{E}_{rad}/dt$ is

$$W(k) = \frac{2}{\pi|k|}\mathrm{Re}\{r^*(k)\, dr(k)/dt\}, \tag{44}$$

where $\mathrm{Re}\{\cdots\}$ stands for the real part, so that we need to calculate the reflection coefficient $r(k)$. An equation describing the time evolution of $r(k)$ (derivation see in Appendix B) is

$$\frac{\partial r}{\partial t} - 4i\lambda^4 r = i\lambda\int_{-\infty}^{\infty}(p\bar{\psi}_{2,s}^2 + p^*\bar{\psi}_{1,s}^2)dz, \tag{45}$$

where $\bar{\psi}_{1,s}$ and $\bar{\psi}_{2,s}$ are one-soliton Jost solutions defined by Eqs. (A2) and (A3), respectively.

Consider first the collision-dominated case, when the perturbation p in Eq. (45) is given by the diffusive term, Eq. (37). If $p=0$, the time evolution of the reflection coefficient r is given by Eq. (14). The right-hand side of Eq. (45) describes the influence of the perturbation on r in the presence of the soliton. As was said above, in the linear limit (vanishing nonlinearity and no solitons) r/λ is a Fourier transform of $u^*(z,t)$ and we insert a diffusive term $\Gamma = Dk^2$ in the left-hand side of Eq. (45) in order to take into account a linear damping of the quasilinear waves. Then, Eq. (45) becomes

$$\frac{\partial r}{\partial t} + \Gamma(\lambda)r - 4i\lambda^4 r = e^{-i\Omega(\lambda)t + 4i\lambda^4 t}F(\lambda), \tag{46}$$

where $\Omega(\lambda) = 8(\lambda^4 + \eta^2)$, $\Gamma(\lambda) = 4D\lambda^4$, and $F(\lambda)$ is some function, which can be written in explicit form after calculating the integrals in Eq. (45). The expression for $F(\lambda)$ for the general soliton solution, Eq. (17), turns out to be too complicated, so that we restrict ourselves to the case when $\xi = 0$ (motionless soliton in the frame moving with the Alfvén velocity v_A). In this case one can get for F the expression

$$F(\lambda) = 4\pi D\lambda\eta^{3/2}\frac{\mu(1+\mu^2)(e^{\pi\mu/4} - \mu e^{-\pi\mu/4})}{(\mu - i)^2\cosh(\pi\mu/2)}, \tag{47}$$

where $\mu = \lambda^2/\eta$.

If $r(\lambda,0)=0$ at the initial time $t=0$ (i.e., one pure soliton), then the solution of Eq. (46) is

$$r(\lambda,t) = \frac{F(\lambda)}{\Gamma(\lambda) - i\Omega(\lambda)}e^{4i\lambda^4 t}(e^{-i\Omega(\lambda)t} - e^{-\Gamma(\lambda)t}). \tag{48}$$

Thus, in the quasistationary regime we have, for the reflection coefficient,

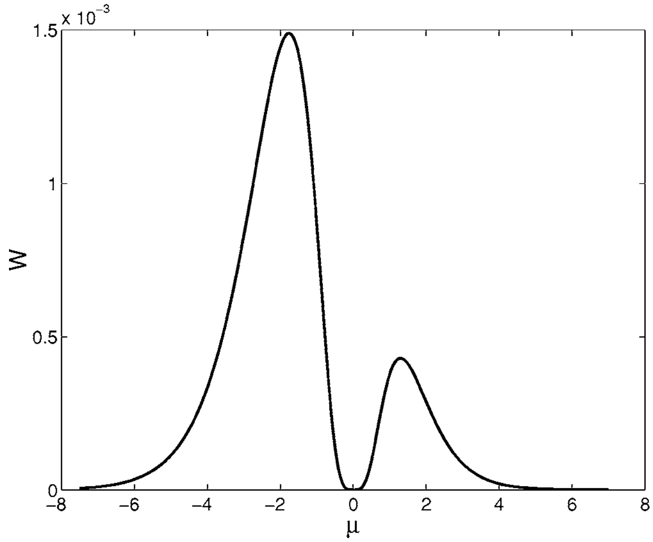

FIG. 1. Spectral density of power emitted by the soliton for $D=0.1$ and $k_0=2$.

$$r(\lambda,t)=\frac{F(\lambda)e^{-i\Omega(\lambda)t+4i\lambda^4 t}}{\Gamma(\lambda)-i\Omega(\lambda)} \tag{49}$$

and, therefore,

$$\mathrm{Re}\left(r^*\frac{dr}{dt}\right)=\frac{\Gamma|F|^2}{\Gamma^2+\Omega^2}. \tag{50}$$

Using Eqs. (44), (47), and (50), we find a final expression for spectral density of power emitted by the soliton in the presence of dissipative perturbation of the form Eq. (37):

$$W(k)=\frac{2\pi D^3 k_0\mu^4\left(\mu e^{-\pi\mu/4}-e^{\pi\mu/4}\right)^2}{[D^2\mu^4+4(1+\mu^2)^2]\cosh^2(\pi\mu/2)}, \tag{51}$$

where the parameter $\mu=k/k_0$ (the normalized wave number) is the ratio of the soliton width to the wavelength of the emitted waves. The spectral composition $Q(k)=d\mathcal{K}_{rad}(k)/dt$ of the emitted magnetic helicity rate, as follows from Eq. (34), is

$$Q(k)=-\frac{W(k)}{2k_0\mu}. \tag{52}$$

It is interesting to note that $Q(k)$ is a universal function of only one (for fixed D) dimensionless parameter μ. The spectral distribution $W(k)$ of emitted power is plotted in Fig. 1 for $D=0.1$ and $k_0=2$. The distribution has two asymmetrical peaks and exponentially decaying tails. The left peak is higher than the right one. It should be noted here that this picture is valid only for the right-hand-side polarized soliton and for $s>0$ (i.e., for a plasma with $\beta<1$) in Eq. (1). Since the case $s<0$ can be obtained from $s>0$ by a transformation $z\to -z$, one can see that in a plasma with $\beta>1$ (i.e., when kinetic pressure of plasma is larger than the magnetic one) or for the left-hand-side polarized soliton, the right peak in Fig. 1 becomes higher than the left one.

Having determined the reflection coefficient r, we can calculate the radiative field $u_c(z,t)$ by substituting the one-soliton Jost solutions $\psi_{1,s}$ and $\bar{\psi}_{1,s}$ into the first term of Eq. (36). Making a change of variable $k=k_0\mu$ in the integral in Eq. (36), one can write

$$u_c(z,t)=\frac{k_0}{\pi}\int_{-\infty}^{\infty}(R\psi_{1,s}^2+R^*\bar{\psi}_{1,s}^2)d\mu, \tag{53}$$

where, as follows from Eq. (49),

$$R(\mu)=\frac{\sqrt{2}\pi D\mu(\mu+i)\left(e^{\pi\mu/4}-\mu e^{-\pi\mu/4}\right)e^{-i(2+\mu^2)k_0^2 t}}{\sqrt{k_0}[D\mu^2-2i(1+\mu^2)](\mu-i)\cosh(\pi\mu/2)}. \tag{54}$$

Integral in Eq. (53) can be evaluated with the aid of residues. For example, the function $R\psi_{1,s}^2$ in the upper half plane of the complex variable μ (this corresponds to the region $z>0$) has poles at $\mu=i$ and $\mu=-1+iD/4$ (taking into account that $D\ll 1$). In addition, it has an infinite set of poles $\mu_n=i+2ni$ (where $n=0,1,2,\ldots$), originating from $\cosh(\pi\mu/2)$ in the denominator of Eq. (54). Calculating the residues, one can see that, for the region not too close to $|z|=0$ (more exactly, the relation $|z|>1/k_0$ must be satisfied), the main contribution in Eq. (53) comes from poles with the smallest imaginary parts. The contribution from poles with sufficiently large imaginary parts vanishes rapidly as $|z|>1/k_0$. Then, since $D\ll 1$, one can take into account only the poles containing D in the imaginary parts. Carrying out the calculations, we find that the radiative field in the region $|z|>1/k_0$ and at times $\sim 1/D$ can be estimated as

$$u\sim\frac{\sqrt{2k_0}\pi D}{2\cosh(\pi/2)}e^{-ik_0|z|-k_0D|z|/4}\left\{\frac{A}{\cosh^2(k_0z+i\pi/4)}+B\left[\frac{\cosh(k_0z+i\pi/4)}{\cosh(k_0z-i\pi/4)}-1\right]^2\right\}, \tag{55}$$

where $A=-\cosh(\pi/4)$ and $B=-\sinh(\pi/4)$ for $z>0$ and $A=\sinh(\pi/4)$, $B=-\cosh(\pi/4)$ for $z<0$.

In the general one-soliton case, an exact calculation of integrals in Eq. (45) for a perturbation given by Eq. (41) is not possible, so that we restrict ourselves to the case of the so-called algebraic soliton [8]. The algebraic soliton solution, can be obtained from the solution Eq. (17), by taking the limit $\eta\to 0$ (i.e., the zero λ_1^2 is approaching the real axis) provided that $\xi<0$:

$$u_s=|u_s|e^{i\phi}\left(\frac{1-i\upsilon y}{1+i\upsilon y}\right)^{3/2}, \tag{56}$$

$$|u_s|=4\sqrt{\frac{\xi}{1+16\xi^2y^2}}, \tag{57}$$

$$y=z-\upsilon t-z_0,\quad \phi=\phi_0-2\xi y,\quad \upsilon=-4\xi. \tag{58}$$

This soliton has algebraically decaying tails and, as follows from Eq. (30), the largest ($E_s=4\pi$) energy. The corresponding Jost solutions are given in Appendix A. Substituting Eqs. (41), (56), (A9), and (A10) into Eq. (45) we get Eq. (46) with $\Omega(\lambda)=8\lambda^2(\lambda^2+|\xi|)$ and

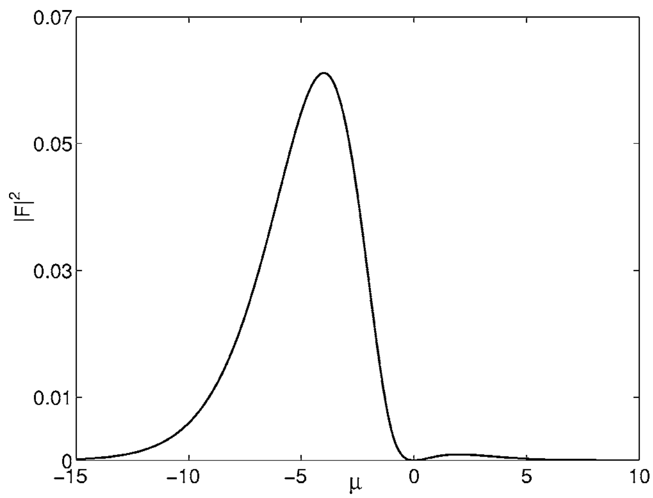

FIG. 2. The shape of the function $|F(\mu)|^2$ in Eq. (62) for $C=0.1$ and $k_0=2$.

$$F(\lambda) = -28\pi C\lambda^3|\xi|^{1/2}e^{-|\alpha|/2}[1+|\alpha|H(-\alpha)], \tag{59}$$

where $\alpha=1+\lambda^2/|\xi|$ and $H(x)$ is Heaviside step function:

$$H(x) = \begin{cases} 0, & x<0, \\ 1, & x\geqslant 0. \end{cases} \tag{60}$$

Since linear damping is absent in this case, we will set in the final answer $\Gamma\to 0$. Denoting $\mu=k/k_0$ with $k_0=2|\xi|$, we have

$$|F|^2 = \frac{49}{16}\pi^2C^2|v|^4|\mu|^3e^{-|\alpha|}[1+|\alpha|H(-\alpha)]^2, \tag{61}$$

where $\alpha=1+\mu$. Since $\Gamma/[\Gamma^2+\Omega^2]\to\pi\delta(\Omega)$ as $\Gamma\to 0$, we find, for the spectral density of emitted power the expression,

$$W(k) = \frac{2|F|^2}{|k|}\delta(2k^2+2kk_0). \tag{62}$$

One can see that the emission is concentrated at two points of the spectrum: $k=0$ and $k=-k_0$. The plot of the function $|F(\mu)|^2$ is presented in Fig. 2.

V. INFLUENCE OF FLUCTUATIONS OF THE PLASMA DENSITY

In this section we consider the influence of random fluctuation of the equilibrium plasma density on the Alfvén soliton. As in previous section, we are interested in radiative effects.

The more general form of the nonlinear term in Eq. (1) is $i\partial_z(nu)$, where n is the plasma density perturbation. In such form Eq. (1) is valid, for example, for the case when Alfvén waves couple strongly with magnetoacoustic modes and there is an independent evolution equation [23,24] for n. The static case $n=-|u|^2$ corresponds to the DNLS equation (1). In the presence of fluctuations of the plasma density we represent (for a given realization) the density as $n\to n+f$, where f stands for the random part. Under this, in the static case the perturbation term in Eq. (1) takes the form

$$p = i\frac{\partial}{\partial z}(fu), \tag{63}$$

where $f(z,t)$ is assumed to be a real Gaussian homogeneous random field with the zero average $\langle f\rangle=0$ and the correlator

$$\langle f(z,t)f(z',t')\rangle = D(z-z')B(t-t'), \tag{64}$$

where the angular brackets denote ensemble averaging.

Substituting Eq. (63) into Eq. (45) and using Eq. (3) one can obtain for the function $\rho(\lambda)=r(\lambda)\exp(-4i\lambda^4t)$ the equation

$$\frac{d\rho}{dt} = -2i\lambda e^{-4i\lambda^4t}\int_{-\infty}^{\infty}f(z,t)G(\lambda,z,t)dz, \tag{65}$$

where

$$G(\lambda,z,t) = \lambda^2(u_s\bar{\psi}_{2,s}^2+u_s^*\bar{\psi}_{1,s}^2)+2i\lambda|u_s|^2\bar{\psi}_{1,s}\bar{\psi}_{2,s}. \tag{66}$$

Multiplying the right-hand side of Eq. (65) by $\exp(\epsilon t)$ with an infinitely small $\epsilon>0$ (as usual, this implies adiabatically turning on a perturbation that was absent at $t=-\infty$) and integrating, we get

$$\rho = -2i\lambda\int_{-\infty}^{t}\int_{-\infty}^{\infty}e^{-4i\lambda^4\tau+\epsilon\tau}f(z,\tau)G(\lambda,z,\tau)d\tau dz. \tag{67}$$

Multiplying Eq. (65) by the complex-conjugate expression (67) and averaging yields

$$\left\langle r^*\frac{dr}{dt}\right\rangle = 4|\lambda|^2\int_{-\infty}^{t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{4i\lambda^4(\tau-t)+\epsilon\tau}B(t-\tau) \times D(z-z')G(\lambda,z,t)G^*(\lambda,z',\tau)d\tau dz dz'. \tag{68}$$

Introducing Fourier transforms of the time and space correlators $B(t)$ and $D(x)$ through

$$B(t) = \int_{-\infty}^{\infty}\tilde{B}(\omega)\exp(-i\omega t)d\omega, \tag{69}$$

$$D(z) = \int_{-\infty}^{\infty}\tilde{D}(q)\exp(-iqz)dq, \tag{70}$$

calculating integrals over z and z' in Eq. (68), we can perform then the integration over τ in Eq. (68) and, after lengthy but straightforward calculations, obtain

$$\left\langle r^*\frac{dr}{dt}\right\rangle = \frac{16\pi^2\eta^2|\mu|\mu^2}{[1+(\mu+\nu)^2]^2\sqrt{1+\nu^2}} \times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{i\tilde{B}(\omega)\tilde{D}(q)I^2(q)}{(h-\omega-qv+i\epsilon)\cosh^2(\pi\alpha/2)}d\omega dq, \tag{71}$$

where we have introduced the notations

$$h = -2k_0^2(1+\mu^2)-\mu k_0v, \tag{72}$$

$$\nu = -\xi/\eta, \quad \alpha = \mu+\nu+q/k_0, \tag{73}$$

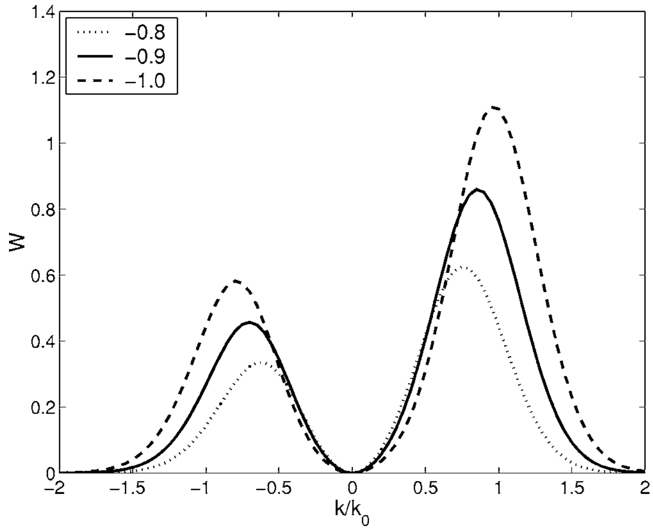

FIG. 3. Spectral density of the emitted power for different values of ν and for $D_0=0.01$, $k_0=1$, and $q_c=5$.

$$I(q) = \sqrt{1+\nu^2}(5q/k_0 - 8\nu - 2\mu)e^{\theta\alpha} - [3\mu q/k_0 + 2\mu^2 + 4(1+\nu^2)]e^{-\theta\alpha}. \tag{74}$$

Then, making use of the relation $\lim_{\epsilon\to 0}(y-i\epsilon)^{-1}=P(1/y)+i\pi\delta(y)$, where P is the symbol of the principal value, one can find

$$\mathrm{Re}\left\langle r^*\frac{dr}{dt}\right\rangle = \frac{16\pi^3\eta^2|\mu|\mu^2}{[1+(\mu+\nu)^2]^2\sqrt{1+\nu^2}} \times \int_{-\infty}^{\infty}\frac{\widetilde{B}(h-qv)\widetilde{D}(q)I^2(q)}{\cosh^2(\pi\alpha/2)}dq. \tag{75}$$

Substituting Eq. (75) into Eq. (44) gives the averaged power spectral density $\langle W(k)\rangle$ emitted by the soliton.

Consider first the case when $f(z,t)$ is a random function of z only (space irregularities), so that we have $\widetilde{B}(\omega)=\delta(\omega)$. In this case the averaged emission power spectral density is

$$\langle W(k)\rangle = \frac{8\pi^2 k_0\mu^2\widetilde{D}(h/v)I^2(h/v)}{[1+(\mu+\nu)^2]^2\sqrt{1+\nu^2}\cosh^2[\pi(\nu^2-\mu^2-1)/2\nu]}, \tag{76}$$

which can be explicitly written for the arbitrary form of the correlator $\widetilde{D}(q)$. In particular, in the case of a noise spectrum of Lorenzian shape we have

$$\widetilde{D}(q) = \frac{D_0 q_c}{\pi[(q-q_0)^2+q_c^2]}, \tag{77}$$

where D_0 is the integral intensity of the noise. The spectral composition of emitted power is shown in Fig. 3 for different values of $\nu=-\eta/\xi$ (with $q_c=5$, $k_0=1$, $D_0=0.01$).

Consider now the case when the space part $\varepsilon(z)$ of the random function $f(z,t)$ has the form

$$\varepsilon(z) = \varepsilon_0\cos(q_0 z + \vartheta), \tag{78}$$

where the random amplitude ε_0 is a zero-mean, normally distributed value with variance σ^2 and the random phase ϑ is uniformly distributed between 0 and 2π. The correlation function of such a process is $D(z)=(\sigma^2/2)\cos(q_0 z)$ or, in the wave number domain,

$$\widetilde{D}(q) = \frac{\sigma^2}{4}[\delta(q-q_0)+\delta(q+q_0)]. \tag{79}$$

In this case the space noise has an infinite correlation length and is concentrated at the wave number q_0. Then, the spectral density of the emitted power is

$$\langle W(k)\rangle = \frac{4\pi^3\eta^2|\mu|\mu^2\sigma^2}{[1+(\mu+\nu)^2]^2\sqrt{1+\nu^2}} \times \left\{\frac{\widetilde{B}(h-q_0 v)I^2(q_0)}{\cosh^2(\pi\alpha_+/2)} + \frac{\widetilde{B}(h+q_0 v)I^2(-q_0)}{\cosh^2(\pi\alpha_-/2)}\right\}, \tag{80}$$

where $\alpha_\pm=\mu+\nu\pm q_0/k_0$, and can be written in a closed form for the arbitrary frequency correlator $\widetilde{B}(\omega)$. It follows from Eq. (80) that in the case when only space irregularities are present, the emission is concentrated at four points of the spectrum:

$$k_{1,2} = [-v \pm \sqrt{v^2 - 8(2k_0^2 + q_0 v)}]/4 \tag{81}$$

and

$$k_{3,4} = [-v \pm \sqrt{v^2 - 8[(2k_0^2 - q_0 v)}]/4. \tag{82}$$

One can see that for the points $k_{1,2}$ the emission takes place provided that the soliton velocity satisfies the conditions $v > 4(q_0+\sqrt{q_0^2+k_0^2})$ or $v < 4(q_0-\sqrt{q_0^2+k_0^2})$. For the points $k_{3,4}$ the corresponding conditions have the same form except that q_0 changes its sign.

VI. CONCLUSION

In this paper we have applied soliton perturbation theory based on the inverse scattering transform method to the study of radiative effects which arise under the action of perturbations on the Alfvén soliton. We have derived equations describing the evolution of the continuous spectrum scattering data. Several forms of perturbations have been considered, including the finite electric conductivity, the effect of resonant particles (nonlinear Landau damping), and the random inhomogeneity of the plasma density. Spectral distributions (in the wave number domain) of the emitted energy and magnetic helicity rates have been calculated analytically. For a perturbation in the form of the finite electric conductivity we also determined the space structure of the radiative field.

Note that numerous satellite observations of the magnetic activity in the solar wind plasma have exhibited the nonlinear nature of Alfvén waves [1,26,27]. In spite of its simplicity, the DNLS equation, being corrected with additional terms, seems to be quite adequate for modeling weakly nonlinear Alfvén waves in space plasma, including solitons

[21,22]. Though the quantitative agreement between observations and theory still needs considerable efforts, the results of the present paper—namely, the asymmetry of the energy spectral distribution—may be used for identification of the left- and right-hand-side polarized solitons in space plasma observations. Furthermore, new observations in space plasma could be conducted to investigate the possible connection between the structure of the soliton radiative tails and rotational discontinuities at the edge of the nonlinear Alfvén waves [26,27].

APPENDIX A: ONE SOLITON SCATTERING DATA AND JOST SOLUTIONS

The one-soliton scattering data are

$$a(\lambda)=\frac{\lambda_1^{*2}(\lambda^2-\lambda_1^2)}{\lambda_1^2(\lambda^2-\lambda_1^{*2})},\quad b(\lambda)=0,\quad \lambda_1^2=\xi+i\eta. \tag{A1}$$

The one-soliton Jost solutions are [25]

$$\bar{\psi}_{1,s}=\frac{\lambda_1 e^{-i\lambda^2 z-2i\lambda^4 t}}{\lambda_1^*(\lambda^2-\lambda_1^2)}\left[\lambda^2 A_2^*(z,t)-|\lambda_1|^2\right], \tag{A2}$$

$$\bar{\psi}_{2,s}=-\frac{\lambda_1\lambda e^{-i\lambda^2 z-2i\lambda^4 t}}{\lambda_1^*(\lambda^2-\lambda_1^2)}A_1^*(z,t), \tag{A3}$$

where

$$A_1(z,t)=\frac{i\eta}{|\lambda_1|}\frac{\exp(i\phi)}{\cosh(k_0 y+i\theta)}, \tag{A4}$$

$$A_2(z,t)=\frac{\cosh(k_0 y+i\theta)}{\cosh(k_0 y-i\theta)}. \tag{A5}$$

Here, ϕ, k_0, y, and θ are the same as in Eqs. (18) and (19). Under this,

$$u_s(z,t)=\frac{2iA_1(z,t)}{A_2(z,t)}. \tag{A6}$$

The expressions for the Jost solutions φ_s, $\bar{\varphi}_s$, and ψ_s can be obtained from Eqs. (A2) and (A3) with the aid of Eqs. (9), (10), and (12).

For the algebraic soliton ($\eta\to 0$, $\xi<0$) we have

$$\theta=\pi/2,\quad \lambda_1^2=\xi,\quad v=-4\xi, \tag{A7}$$

$$y=z-vt-z_0,\quad \phi=\phi_0-2\xi y, \tag{A8}$$

and the corresponding Jost solutions are

$$\bar{\psi}_{1,s}=-\frac{e^{-i\lambda^2 z-2i\lambda^4 t}}{(\lambda^2-\xi)}\left[\lambda^2 A_2^*(z,t)-|\xi|\right], \tag{A9}$$

$$\bar{\psi}_{2,s}=\frac{\lambda e^{-i\lambda^2 z-2i\lambda^4 t}}{(\lambda^2-\xi)}A_1^*(z,t), \tag{A10}$$

where

$$A_1(z,t)=\frac{2i\sqrt{-\xi}\exp(i\phi)}{(1-4i\xi y)}, \tag{A11}$$

$$A_2(z,t)=\frac{(1-4i\xi y)}{(1+4i\xi y)}. \tag{A12}$$

APPENDIX B: DERIVATION OF PERTURBATION THEORY EQUATIONS

Equations describing the evolution of the scattering data in the presence of perturbations for the DNLS equations were first obtained by Wyller and Mjølhus [11]. Since a detailed derivation of these equations was absent in Ref. [11], we present the derivation following Ref. [25].

Equation (1) can be cast in the matrix form

$$\partial_t U-\partial_z V+[U,V]+P=0, \tag{B1}$$

where

$$P=\begin{pmatrix}0 & i\lambda p\\ -i\lambda p^* & 0\end{pmatrix}. \tag{B2}$$

From Eq. (B1) and the fact that $M^{\pm}$ satisfies Eq. (3) one can get

$$(\partial_z-U)(\partial_t-V)M^{\pm}+PM^{\pm}=0. \tag{B3}$$

Introducing a new unknown $J^{\pm}(z,t,\lambda)$ defined through the relation

$$(\partial_t-V)M^{\pm}=M^{\pm}J^{\pm}, \tag{B4}$$

one can obtain that $J^{\pm}$ satisfies $\partial_z J^{\pm}=-M^{\pm-1}PM^{\pm}$ and, therefore, $J^{\pm}=C^{\pm}-\int_{\pm\infty}^{z}M^{\pm\,-1}PM^{\pm}dz'$, where the constant matrices $C^{\pm}$ are determined from the boundary conditions at $x\to\pm\infty$. Since $V=-2i\lambda^4\sigma_3$ as $z\to-\pm\infty$, we have, from Eq. (B4), $C^{\pm}=2i\lambda^4\sigma_3$ and, hence, the following equations of motion for $M^{\pm}$:

$$(\partial_t-V)M^{\pm}=M^{\pm}\left[2i\lambda^4\sigma_3-\int_{\pm\infty}^{z}(M^{\pm})^{-1}PM^{\pm}dz'\right]. \tag{B5}$$

Equation (B5) is valid only for $\mathrm{Im}\,\lambda^2=0$. Introducing the matrix $M(z,t,\lambda)=(\varphi,\psi)$, columns of which ($\varphi$ and ψ) admit analytical continuation to $\mathrm{Im}\,\lambda^2>0$, and as before defining the new unknown matrix $J(z,t,\lambda)=(J_1,J_2)$ through the relation

$$(\partial_t-V)M=MJ, \tag{B6}$$

one can similarly obtain

$$J_1=\begin{pmatrix}2i\lambda^4\\ 0\end{pmatrix}-\int_{-\infty}^{z}M^{-1}P\varphi dz', \tag{B7}$$

$$J_2=\begin{pmatrix}0\\ -2i\lambda^4\end{pmatrix}+\int_{z}^{\infty}M^{-1}P\psi dz'. \tag{B8}$$

Thus, we have the equations of motion valid for $\mathrm{Im}\,\lambda^2>0$ except at λ_j, where M fails to be invertible. Making the natural assumption that the zeros $\lambda=\lambda_j$ are simple, one can show (see below) that each singularity is removable since

$\det M = a$. Differentiating Eq. (8) with respect to t and using Eq. (B5) yields

$$\partial_t S(t,\lambda) + 2i\lambda^4[\sigma_3, S(t,\lambda)] = -\int_{-\infty}^{\infty} (M^+)^{-1}(z,t,\lambda) P M^-(z,t,\lambda) dz. \quad (B9)$$

The equations of motion for the coefficients $a(t,\lambda)$ and $b(t,\lambda)$ are contained in Eq. (B9):

$$\frac{\partial a}{\partial t} = -i\lambda \int_{-\infty}^{\infty} (p\psi_2\varphi_2 + p^*\psi_1\varphi_1) dz, \quad (B10)$$

$$\frac{\partial b}{\partial t} - 4i\lambda^4 b = i\lambda \int_{-\infty}^{\infty} (p\bar{\psi}_2\varphi_2 + p^*\bar{\psi}_1\varphi_1) dz. \quad (B11)$$

The expression defining the zeros $\lambda_j(t)$ of $a(t,\lambda)$ is $a(t,\lambda_j(t))=0$. Differentiating with respect to t gives

$$\partial_t a(t,\lambda_j(t)) + \frac{\partial \lambda_j}{\partial t} a_j' = 0, \quad (B12)$$

where $a_j' = da/d\lambda|_{\lambda=\lambda_j}$. Using Eqs. (B10) and (B12) we have

$$\frac{\partial \lambda_j^2}{\partial t} = \frac{2i\lambda_j^2}{a_j'} \int_{-\infty}^{\infty} (p\psi_{2,j}\varphi_{2,j} + p^*\psi_{1,j}\varphi_{1,j}) dz, \quad (B13)$$

where $\psi_{2,j}$, $\varphi_{2,j}$, $\psi_{1,j}$, and $\varphi_{1,j}$ are the corresponding Jost solutions evaluated at $\lambda=\lambda_j$. To obtain the evolution equation for b_j, we differentiate Eq. (13) with respect to t, use Eqs. (B6), (B7), and (B8), and take the limit $\lambda \to \lambda_j$ applying [since $\det M(\lambda_j) = a(\lambda_j) = 0$] the l'Hopitale rule and using again Eq. (13). As a result, one obtains

$$\frac{\partial b_j}{\partial t} - 4i\lambda^4 b_j = \frac{i\lambda_j}{a_j'} \int_{-\infty}^{\infty} \left\{ p\varphi_2 \frac{\partial}{\partial\lambda}(\varphi_2 - b_j\psi_2) + p^*\varphi_1 \frac{\partial}{\partial\lambda}(\varphi_1 - b_j\psi_1) \right\} dz', \quad (B14)$$

where, after differentiating, the integrand is evaluated at $\lambda=\lambda_j$. Equations (B10), (B11), (B13), and (B14) describe the evolution of the scattering data.

If $p[u,u^*]$ is a small perturbation, one can substitute the unperturbed N-soliton solutions ψ, $\bar{\psi}$, φ, and $\bar{\varphi}$ (for a convenient way for writing down these solutions see [25]) into the right-hand side of Eqs. (B10), (B11), (B13), and (B14). This yields evolution equations for the scattering data in the lowest approximation of perturbation theory. This procedure can be iterated to yield higher orders of perturbation theory. The appearing hierarchy of equations is applied to an arbitrary number of solitons and, in particular, describes nontrivial many-soliton effects in the presence of perturbations. In this paper we restrict ourselves to the case of one-soliton solutions and substitute Eqs. (A2) and (A3) into the right-hand side of Eqs. (B10), (B11), (B13), and (B14). Taking into account that in the zero approximation $\partial a/\partial t = 0$ and $\varphi_s = a\bar{\psi}_s$, from Eqs. (B10) and (B11) we obtain the equation, Eq. (45), for the reflection coefficient $r=b/a$.

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